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### ABSTRACT

A simple frequency transformation method to obtain propagation constant and mode coupling coefficients of a helix waveguide is introduced. Theoretical  $TE_{01}$  mode losses agree well with measured results. Also the transmission characteristics of 5,700 of helix waveguide recently manufactured are described. The inner diameters of them are 51 mm and 60 mm and their unit length is 5 meters.

### Introduction

In order to analyze transmission characteristics of a helix waveguide, it is imperative to know the propagation constants of normal modes in the waveguide and the coupling coefficients between  $TE_{01}$  mode and other normal modes, when the waveguide has mechanical imperfections such as curvature or deformation.

H.G. Unger studied those areas in [1] in 1961. However, not many research works have been embarked upon since then, because it was not easy to get eigenvalues from the characteristic equation of helix waveguide. In this paper, we propose to obtain the propagation constants and coupling coefficients by a simple frequency transformation method using the charts given in [1].

Beyond theoretical study, we have actually manufactured 300 pieces of 51 mm ID and 5,400 pieces of 60 mm ID helix waveguides since 1971. The various characteristics of those waveguides are dealt with in this paper.

### Structure and Manufacturing Technique

The structure of a helix waveguide is shown in Fig. 1. The unit length is 5 meters. A helix waveguide is made by winding insulated copper wire on a manarel, wrapping carbonized plastic tape and glass tape over it, and then inserting the mandrel in a steel pipe and impregnating epoxy resin to form laminated layers of dielectric tapes between the helix and the pipe by the vacuum impregnation method. After curing, the mandrel is taken out.

### Propagation Constant and Coupling Coefficient

The propagation constants of normal modes in helix waveguide and coupling coefficients between  $TE_{01}$  mode and other normal modes are obtainable by a simple method, when the frequency characteristics of wall impedance is known and the charts showing the relationships between the wall impedance and propagation constants at a certain frequency and between the wall impedance and the coupling coefficients at a certain frequency are available.

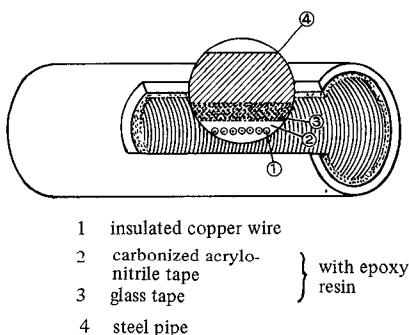


Fig. 1 Structure of helix waveguide

### Propagation Constants of Normal Modes

The wall impedance is a function of frequency and the relationship between the propagation constant  $\gamma_n (= a_n + j\beta_n)$  in helix waveguide and its wall impedance may be shown as<sup>(1)</sup>

$$jka Z/Z_0 = g(\xi_n) \quad (1)$$

$$(\gamma_n a)^2 = \xi_n^2 - (ka)^2$$

where,

$$g(\xi_n) = -\frac{J_p(\xi_n) \{ \xi_n J_{p-1}(\xi_n) - p J_p(\xi_n) \}}{J_{p-1}(\xi_n) \cdot J_{p+1}(\xi_n)}$$

$Z$  : wall impedance  
 $Z_0$  : impedance in free space  
 $k$  : phase constant in free space  
 $a$  : radius of waveguide  
 $p$  : circumferential order  
 $\xi_n$  : eigenvalue of  $n$  mode  
 $J_p$  : first kind Bessel function of  $P$ th order

when we assume  $|Z/Z_0| \ll \frac{1}{p}(ka)$ ,  $g(\xi_n)$  is not an explicit function of frequency. If we assume  $|\xi_n/(ka)| \ll 1$ , then

$$\gamma_n a = \sqrt{\xi_n^2 - (ka)^2} = jka \left\{ 1 - \frac{1}{2} \left( \frac{\xi_n}{ka} \right)^2 \right\} \quad (2)$$

Let us assume that the eigenvalue of  $TE_{01}$  mode is  $\xi_{01}$  in a perfect copper waveguide, then we get from Equation (2)

$$\text{Im}(\xi_n)^2 = 2(ka)(a_n a) \quad (3)$$

$$\text{Re}(\xi_n)^2 = \xi_{01}^2 - 2(ka)(\Delta\beta_n a)$$

where,  $\Delta\beta_n = \beta_n - \beta_{01}$

Let  $Z_1$  and  $(\gamma_n)_1$  be the wall impedance and the propagation constant at frequency  $f_1$  respectively and assume we have a chart which shows the relationship between  $Z_1/Z_0$  and  $(\gamma_n)_1$  and we know the frequency characteristics of the wall impedance  $Z$ , then we can get the relationship between  $Z/Z_0$  and  $\gamma_n$  over the same frequency range. That is, let  $Z_2$  be the wall impedance at frequency  $f_2$  and let  $\text{Re}(\xi_n)$  and  $\text{Im}(\xi_n)$  stay constant, then  $(ka)_2(a_n a)_2$  and  $(ka)_2(\Delta\beta_n a)_2$  can be obtained uniquely because  $\text{Re}(\xi_n) > 0$  and  $\text{Im}(\xi_n) > 0$ . From Equation (3), we get

$$(ka)_1(a_n a)_1 = (ka)_2(a_n a)_2 \quad (4)$$

$$(ka)_1(\Delta\beta_n a)_1 = (ka)_2(\Delta\beta_n a)_2$$

If radius  $a$  is same at both sides of Equation (4), we get

$$(a_n a)_2 = (a_n a)_1 \cdot f_1/f_2 \quad (5)$$

$$(\Delta\beta_n a)_2 = (\Delta\beta_n a)_1 \cdot f_1/f_2$$

On the other hand, Equation (1) at frequency  $f_2$  if written as

$$j(ka)_2 \frac{Z}{Z_0} = j(ka)_1 \frac{Z_2}{Z_0} \frac{(ka)_2}{(ka)_1} = g(\xi_n) \quad (6)$$

If we put

$$Z_1 = Z_2 \frac{(ka)_2}{(ka)_1} = Z_2 \frac{f_2}{f_1} \quad (7)$$

Equation (6) can be written as

$$j(ka)_1 \frac{Z_1}{Z_0} = g(\xi_n) \quad (8)$$

Equation (8) indicates that  $Z_1$  is the wall impedance when frequency is transformed from  $f_2$  to  $f_1$  without changing  $\xi_n$ .

When there is a chart of  $Z_1/Z_0 \sim (\gamma_n a)_1$  at  $f_1$ , the propagation constant  $(\gamma_n)_2$  at frequency  $f_2$  is obtained as follows:

1. Get the wall impedance  $Z_2$  at frequency  $f_2$ .
2. Transform  $Z_2$  to  $Z_1$  at frequency  $f_1$  by using Equation (7). ( $\xi_n$  is same at this time)
3. Read out the values of  $(a_n a)_1$  and  $(\Delta \beta_n a)_1$  by the chart of  $Z_1/Z_0 \sim (\gamma_n a)_1$ . The chart of  $Z_1/Z_0 \sim (\gamma_n a)_1$  is shown in [1] in case of  $a = 25.5$  mm and  $f_1 = 55.5$  GHz.
4. Obtain  $(\gamma_n a)_2$ ,  $(\Delta \beta_n a)_2$  by Equation (5).

### Coupling Coefficients

When a waveguide has curvature or deformation, mode conversions occur among normal modes. The coefficients of coupling due to curvature between  $TE_{01}$  mode and other normal modes are given in [1]. By modifying them, we can get the following equation,

$$RC_{n[01]}^R = A^R(\xi_n)(ka) \quad (9)$$

where,

$$A^R(\xi_n) = N_n \cdot \pi/2 \cdot J_1(\xi_n) \frac{-4\xi_{01}\xi_n}{(\xi_n^2 - \xi_{01}^2)^2} G_n$$

$C_{n[01]}^R$  : coupling coefficient  
 $R$  : radius of curvature  
 $N_n$  : normalization factor  
 $G_n = J_1(\xi_n)/\xi_n J'_1(\xi_n)$

$A^R(\xi_n)$  is not an explicit function of frequency. Equation (9) is deduced from the assumption that

$$|\xi_n| \ll ka \text{ and } |2 G_n(ka)^2 / (\xi_n^2 - \xi_{01}^2)| \gg 1$$

These conditions are not uncommon in practice. From Equation (9), the following relation is derivable for  $(RC_{n[01]}^R)_1$  at frequency  $f_1$  and  $(RC_{n[01]}^R)_2$  at  $f_2$

$$(RC_{n[01]}^R)_2 = (RC_{n[01]}^R)_1 \cdot \frac{f_2}{f_1} \quad (10)$$

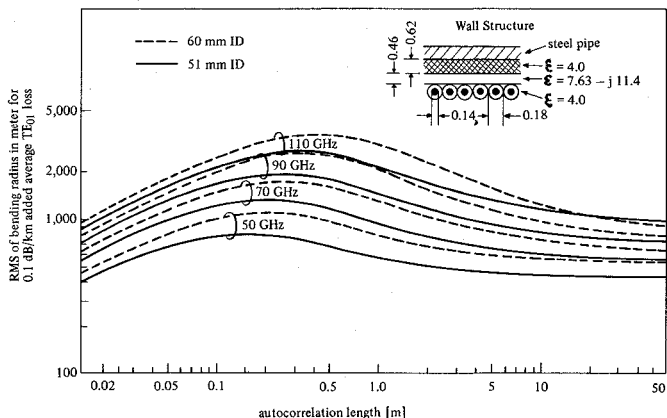


Fig. 2  $TE_{01}$  loss in helix waveguide with random curvature

When there is a chart that shows the relationship between wall impedance  $Z_1/Z_0$  and coupling coefficient  $(RC_{n[01]}^R)_1$  at frequency  $f_1$ , the coupling coefficient  $(RC_{n[01]}^R)_2$  at frequency  $f_2$  can be obtained by a similar procedure to the case of the propagation constant in the previous section, using Equations (7), (10) and a chart. The chart of  $Z_1/Z_0 \sim (RC_{n[01]}^R)_1$  is also shown in [1] in case of  $a = 25.5$  mm and  $f_1 = 55.5$  GHz.

The methods described above and in the preceding section can be applied to a waveguide having a different radius from that of waveguide on the chart, since frequency and radius always appear in the form of  $ka$ . A similar method may be used to get the coupling coefficient where deformation of waveguide exists.

### Application

The methods discussed in the previous two sections enable us to calculate the  $TE_{01}$  mode loss due to random curvature of helix waveguide. Fig. 2 shows RMS of radius of curvature that causes an additional 0.1 dB/km loss for the helix waveguides whose inner diameters are 51 mm and 60 mm. The wall structure and parameters necessary for getting its wall impedance are also shown in the figure. The considered modes that couple with  $TE_{01}$  Mode are  $TE_{11}$ ,  $TE_{12}$  and  $TM_{11}$ . The loss is inversely proportional to the square of the curvature.

### $TE_{01}$ Mode Attenuation

The losses of  $TE_{01}$  mode are caused by random curvature of waveguides, deformations of the cross section such as diameter change and ellipticity and imperfections at joints such as tilts and offsets. Fig. 3 and Fig. 4 show theoretical losses of helix waveguides of 51 mm ID and 60 mm ID under the following conditions:

1. RMS of curvature in manufacturing is 1,000 m and its auto-correlation length is 0.1 m.
2. RMS of curvature in installation is 500 m and its auto-correlation length is 2 m.
3. Tilt is 0.5 m rad at every 5 meters.

The contributions to loss of deformations of the cross section and offsets at joints are very small in practice and neglected. The measured average values of RMS of curvature in manufacturing are 1,130 m for 300 pieces of 51 mm ID helix waveguide and 1,460 m for 5,400 pieces of 60 mm ID helix waveguide. Their measured losses are also shown in Fig. 3 and Fig. 4. They agree well with the theoretical results. The attenuation were measured by cascading 30 pieces of the waveguide.

### Attenuation of Unwanted Modes

The conventional method to measure  $TE_{12}$  mode attenuation is the pulse reflecting method using IF amplifier and a movable short plane, because the excitation of  $TE_{12}$  mode is so weak. But we measured the attenuation of  $TE_{11}$  and  $TE_{12}$  mode by directly comparing the amplitude of input signal to that of output through a helix waveguide, because we got a rather strong  $TE_{12}$  mode exciter. The measurement circuit is shown in Fig. 5. The signal was a continuous wave generated by a sweep oscillator. The coupling efficiencies between rectangular  $TE_{10}$  mode and circular  $TE_{12}$  mode of the exciters we used were -1 to -4 dB. The mode discrimination was less than -25 dB. Fig. 6 shows  $TE_{11}$  mode and  $TE_{12}$  mode attenuations over the range from 40 GHz to 90 GHz in 51 mm ID and 60 mm ID helix waveguides.

### Joint

We used coupling sleeves made of iron for jointing waveguide. The structure of a coupling sleeve for 60 mm ID helix waveguide is illustrated in Fig. 7. The tilts and offsets were measured for 540 jointing parts. The RMSs of tilts and offsets were 0.48 mrad and 30  $\mu$ m respectively.

### Reference

1. Unger, H.G. "Normal Modes and Mode Conversion in Helix Waveguide," B.S.T.J., Vol. 40, 1961, P. 255

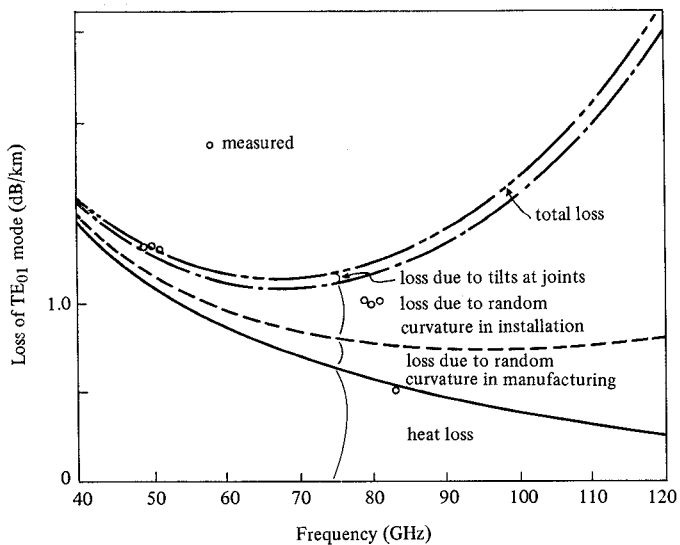


Fig. 3 Loss of TE<sub>01</sub> mode in helix waveguide (51 mm ID)

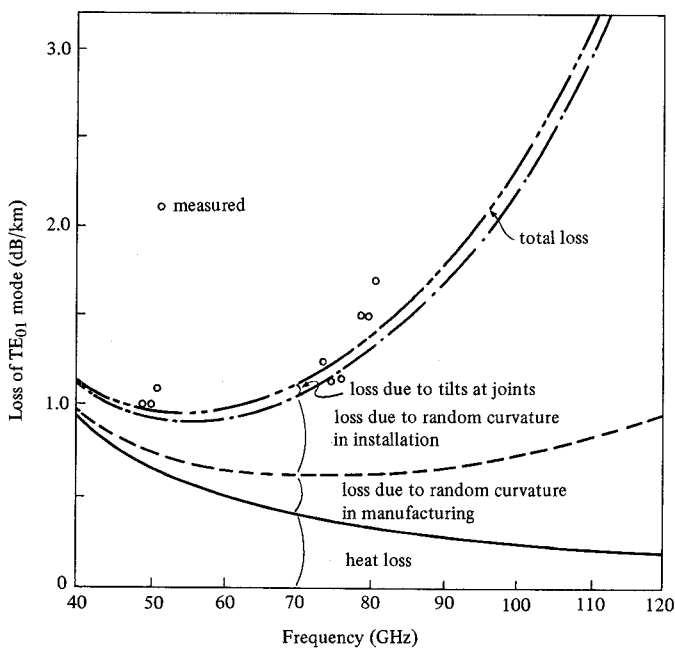


Fig. 4 Loss of TE<sub>01</sub> mode in helix waveguide (60 mm ID)

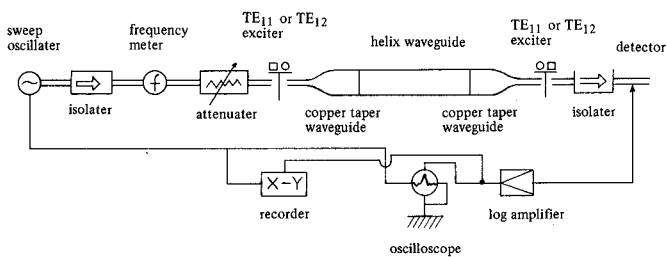


Fig. 5 Measurement Circuit for TE<sub>11</sub> and TE<sub>12</sub> Mode Attenuations

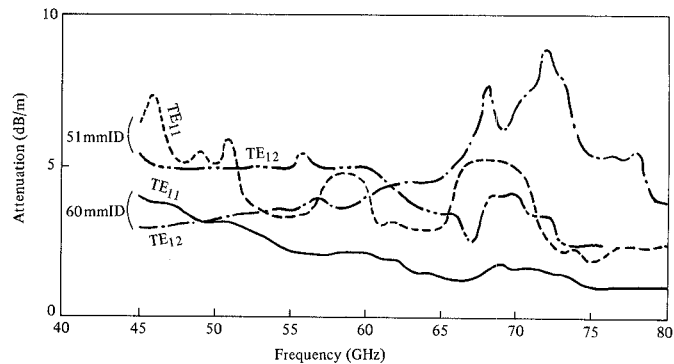


Fig. 6 Attenuation of TE<sub>11</sub> and TE<sub>12</sub> modes (measured)

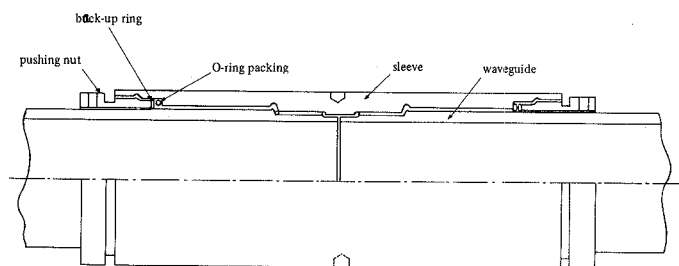


Fig. 7 View of Joint for 60 mm ID helix waveguide